**Special Relativity Postulates**

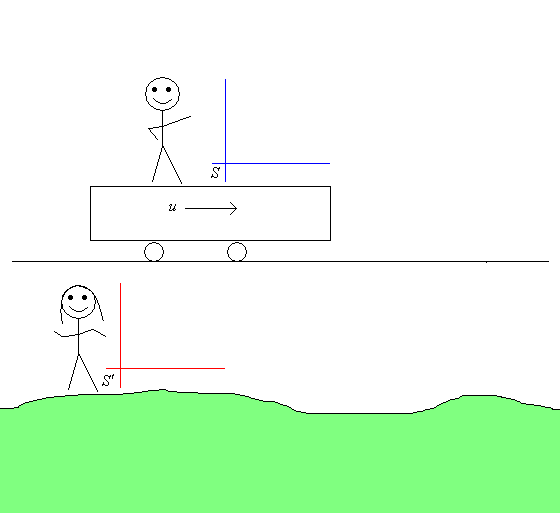
Now we’ll examine the modifications to classical mechanics that are imposed by Einstein’s theory of Relativity. First we’ll consider the special theory. The impetus for this discovery was that ME were not invariant w/r to a Galilean transformation, like N2L was. Therefore it was assumed that they had a special reference frame – like sound waves. But attempts to measure the special reference frame (ether) velocity w/r to the Earth resulted in 0 basically. So there is no ether. This meant that either ME were wrong and they should be Galilean invariant, or N2L was wrong and it shouldn’t be Galilean invariant. Assuming that ME are correct, and that it is N2L which is incorrect, we come to the conclusion that the velocity, c, of the light waves must be invariant w/r to all observers. This can also be argued since the prediction of ME is that light travels with velocity c, but it doesn’t specify a reference frame – so it must be valid for all reference frames. So we make this the cornerstone for developing a new coordinate transformation from S to S′. The actual derivation is pretty straightforward. The brings us to the fundamental postulates of special relativity.



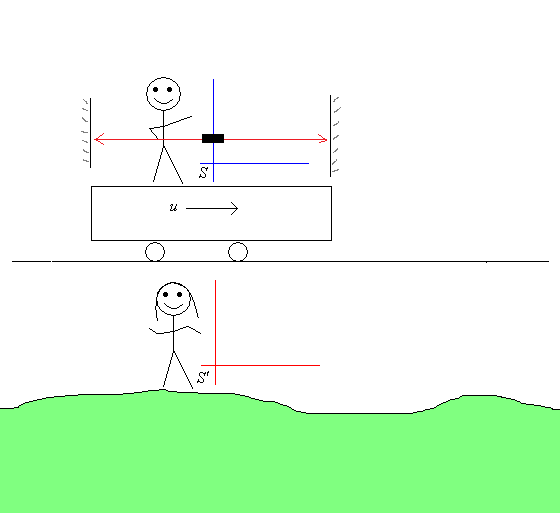
These principles have consequences for our usual concepts of displacement, time, velocity, acceleration, etc. So we’ll first investigate these. First we’ll consider simultaneity.

**Simultaneity**

Consider person on ground, at rest, and another in a train, moving at speed u to the right.



And consider two events which happen at the same time in the reference frame of the person on the train, S. We’d like to consider what time difference elapses between these two events in the frame S´. As an instance of two simultaneous events, consider that the person in S stands in the middle of the train, fixes two mirrors on either end, and turns on a bidirectional light. Then each beam should hit its respective mirror at the same time, according to S. Moreover, let L be the length of the train as measured by S (more on that subtle point later). Then according to S, they will have hit the mirrors simultaneously at t = (L/2c).



What about S´ perception of events? When will they have hit the mirrors according to S´? According to S´, the length of the train is L´ = L/γ where γ = 1/√(1-u2/c2) (need to find a quantitative way to circumvent need for the length contraction formula). So apropos the rightward beam, it starts at coordinate 0, and the right mirror starts at coordinate L´/2. So the right beam’s coordinate as function of time will be 0 + ct (note we are presuming we measure light’s speed to be c in all reference frames, so its velocity will *not* be c+u), and the mirror’s coordinate will be L´/2 + ut´. These two will intersect when,



Apropos the left beam, its position as function of time will be 0-ct´ (again we use relativity postulate to assert the speed of the beam will be c regardless of reference frame, and so its velocity will *not* be -c + u), and the left mirror’s position will be -L´/2 + ut´. These will meet when,



So clearly the left beam will hit first, and the elapsed time would be:



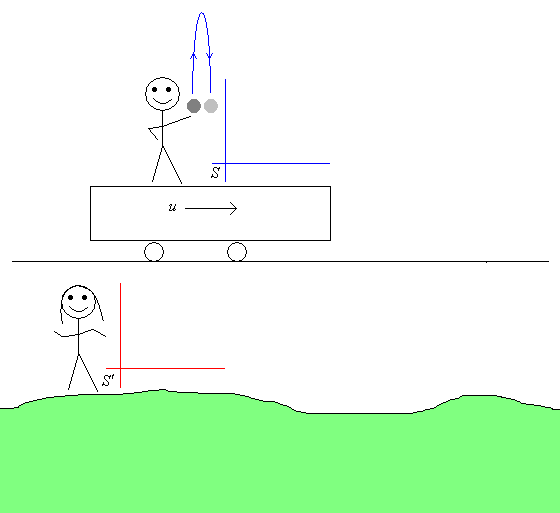
If we fill in L´ = L/γ, we get:



Note that if we hadn’t invoked the relativity postulate to assert the speed of light is c in all reference frames, but was rather c + u when traveling rightward, and -c + u when traveling leftward, then we would’ve gotten t´r,ℓ = L´/2c → Δt´ = 0. But we’ll also observe that our present formula does reduce to Δt´ → 0 in the u → 0 limit, which accords with our classical intuition regarding simultaneity.

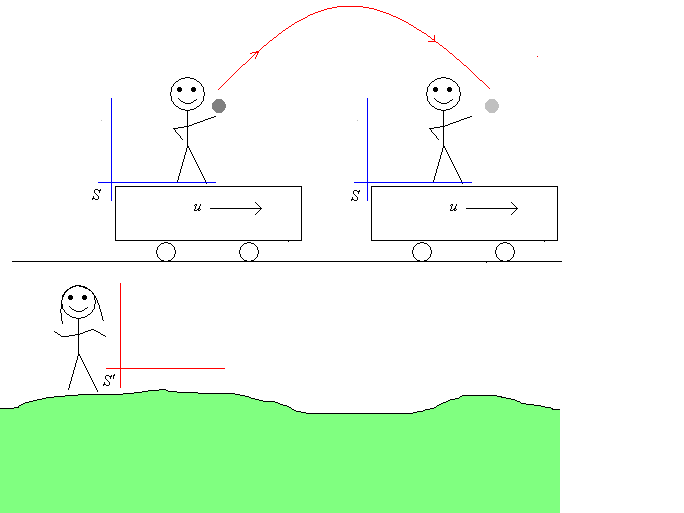
**Temporal distance between two events (time dilation)**

Suppose that two events (Event 1 and Event 2) occur at the same point in space in a certain reference frame. Then the this would be the *proper* frame of reference (S­­) with respect to time for that event. Another way to say it, suppose you have two events that are at rest in your reference frame, i.e., they are not moving and begin and end at the same spot, then this reference frame is the proper reference frame. For instance, consider a person throwing a baseball on a train.



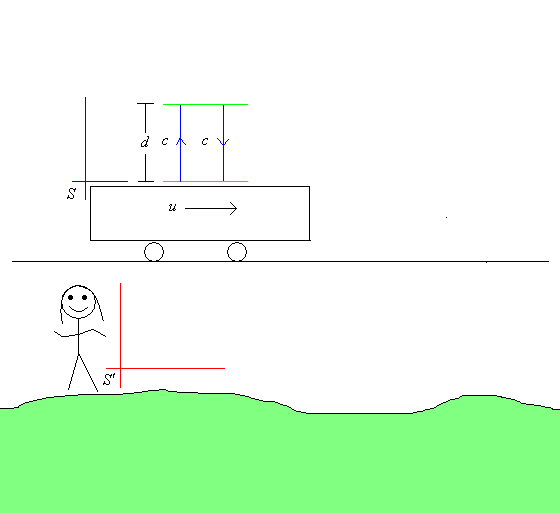
Event 1 would be throwing it, Event 2 would be catching it. And both of these would occur at the same point in the frame of reference of the person on the train.

Now consider an observer on the ground. This person would constitute another inertial frame of reference, S′, which is moving with velocity - u with respect to S. And going back to the baseball example, the two events, catching and throwing the ball will occur at different points in that person’s reference frame. From their vantage point the path of the baseball will look like,



We can show that the time (Δt′) between the two events in the frame S′, that is any inertial reference frame, is longer than the time between the two events in the proper frame S­ (Δt).

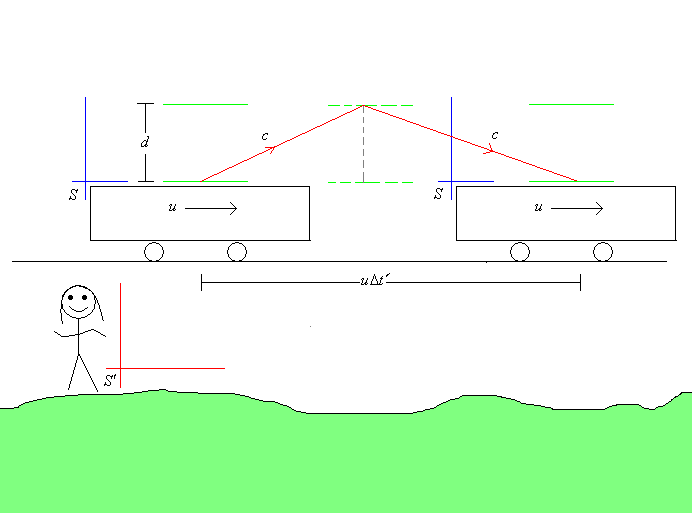
For instance consider the following contrivance. We bounce a ray of light off of a mirror on the train above and determine the length of time required to do this. If we’re on the train then we see light take the following path,



The time would be



But now let us take the vantage point of the person on the ground. To us at rest on the ground, the process will appear to take a time Δt′, which will be longer than Δt. And to us at rest on the ground the ray of light would take the following path during the process.



The time required to make this path would simply the distance light travels divided by its speed. The distance is: , and the speed is still c, since we must always measure the speed of light to be c. Thus the time is:



Now solve for Δt′,



and we can put this in terms of Δt = 2d/c,



defining γ, we can write this as,



Note that time will appear to be dilated in both reference frames (similar to how – Wikipedia analogy) people appear small to each other at a distance in both reference frames.

**Time-dilation for accelerated motion**

Suppose your reference frame, S′, is accelerating with respect to S so that its velocity varies with time according to u(t). If an event lasts a time Δt in reference frame S, what will be the duration of the event in S′? We can apply our formula above at every instant in time over which the velocity is approximately constant. And we’d have:



Adding up all of the time-dilations, i.e. integrating, we’d have



So therefore:



**Example: Mad scientist**

Suppose a mad scientist creates an indestructible bomb which will explode in 2 minutes, and will consequently destroy the entire Earth if it does. And also suppose that we have a really *really* fast car which has a top speed close to the speed of light. How can we save the Earth?

Well, we can put the bomb in the car and accelerate it up to close to the speed of light, say v = 2.9×108 m/s. Then in the bomb’s reference frame it will explode in Δt = 5min. But in our reference frame it will take,



(we might want to accelerate it to a higher velocity in order to live a bit longer).

**Example: Particle decay**

As an example which really does occur, consider a certain type of elementary particle – hadrons. When produced in the lab, they usually decay in 5μs. But when they’re hurtling towards the Earth from the Sun, it takes longer for them to decay - 11.5μs. Why the discrepancy?

Well, in the hadron’s reference frame, the beginning and end of its life takes place at the same point, so its frame of reference constitutes the proper time. So Δt = 5μs. But in our reference frame, we’re moving with a velocity of –v with respect to the hadron (which is the same as saying that it moves with a velocity of v with respect to us) – and so our perception of the lifetime of the hadron will be dilated to an extent depending on v. If we’re at rest w/r to the hadron (i.e. when they’re created in the lab) then v = 0, and γ = 1. Therefore Δt′ = Δt and our perception of the lifetime will be the same as its perception – 5μs. But when its moving with velocity υ, then our frame of reference is moving with velocity –v with respect to the hadron’s, and consequently our perception of its lifetime will dilate. We can determine how fast the hadron must be moving in order for its lifetime to dilate to 11.5μs in our frame of reference. The amount of time it will last in our frame is:



this comes to:



**Example: Twin Paradox**

Another example is the classic twin paradox. If a twin 1 travels at 0.9c for 10 light years and then comes back, she will be aged less than the twin 2 that remained on Earth. Those on Earth will think the trip lasts.



The entire trip is in the proper rest frame of the twin though, and so she will think that the trip lasts,



so this would be another example of time-dilation. The twin that travels in the space ship will really have aged only 5 years, and the Earth-bound one will have aged 11.1 years.

**Example: Accelerated motion**

If a car is accelerating at rate **a** from rest, and an event of duration Δt occurs on it, what would be the duration of the event according to a parked car.

The time duration would be:

